Related Rates #3

1)

$V = \frac{4}{3}\pi r^3$

MOMENT

$r = 5$ ft

RATES

$\frac{dV}{dt} = 100 \text{ ft}^3/\text{min}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$100 = 4\pi(5)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{\pi}$ ft/min

2)

$S = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

MOMENT

$d = 10$ cm

$r = 5$ cm

RATES

$\frac{dS}{dt} = -1 \frac{\text{cm}^2}{\text{min}}$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$-1 = 8\pi(5)\frac{dr}{dt}$

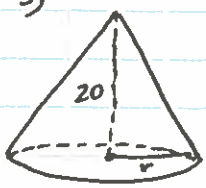
$\frac{dr}{dt} = -\frac{1}{40\pi}$ cm/min

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(5)^2\left(-\frac{1}{40\pi}\right)$

$\frac{dV}{dt} = -\frac{5}{2}$ cm^3/min

3)



$V = \frac{1}{3}\pi r^2(20)$

$V = \frac{20}{3}\pi r^2$

* Note: $h = 20$

MOMENT

$r = 10$ cm

RATES

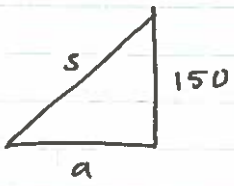
$\frac{dr}{dt} = 2$ cm/sec

$\frac{dV}{dt} = \frac{40}{3}\pi r \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{40}{3}\pi(10)(2)$

$\frac{dV}{dt} = \frac{800\pi}{3}$ cm^3/sec

4)



MOMENT

$s = 250$ ft

$a = 200$ ft

RATES

$\frac{da}{dt} = 20$ ft/sec

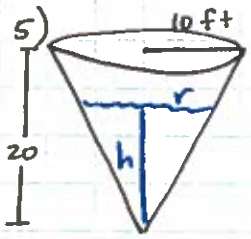
$a^2 + 150^2 = s^2$

$2a\frac{da}{dt} = 2s\frac{ds}{dt}$

$200(20) = 250\left(\frac{ds}{dt}\right)$

$\frac{ds}{dt} = 16$ ft/sec

5)



MOMENT

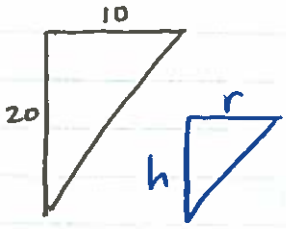
$h = 8$ ft.

RATES

$\frac{dh}{dt} = -\frac{1}{2}$ ft/hr

$V = \frac{1}{3}\pi r^2 h$

Similar Triangles



$\frac{r}{10} = \frac{h}{20}$

$r = \frac{1}{2}h$

Note: Step 4 must be applied in order to have V in terms of h.

Substitute $r = \frac{1}{2}h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 \cdot h$

$V = \frac{1}{3}\pi \cdot \frac{1}{4}h^2 \cdot h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi(8)^2 \cdot \left(-\frac{1}{2}\right)$

$\frac{dV}{dt} = -8\pi$ ft^3/hr.

6)

$S = 4\pi r^2$

MOMENT

$V = \frac{32\pi}{3}\ ft^3$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$\frac{dS}{dt} = 8\pi(2)\left(\frac{1}{4\pi}\right)$

$\frac{dS}{dt} = 4\ \frac{ft^2}{min}$

Rates

$\frac{dV}{dt} = 4\ ft^3/min$

+ need r & $\frac{dr}{dt}$. Use $V = \frac{4}{3}\pi r^3$

$\frac{32\pi}{3} = \frac{4}{3}\pi r^3$

$8 = r^3$

$r = 2$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4 = 4\pi(4)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{4\pi}$